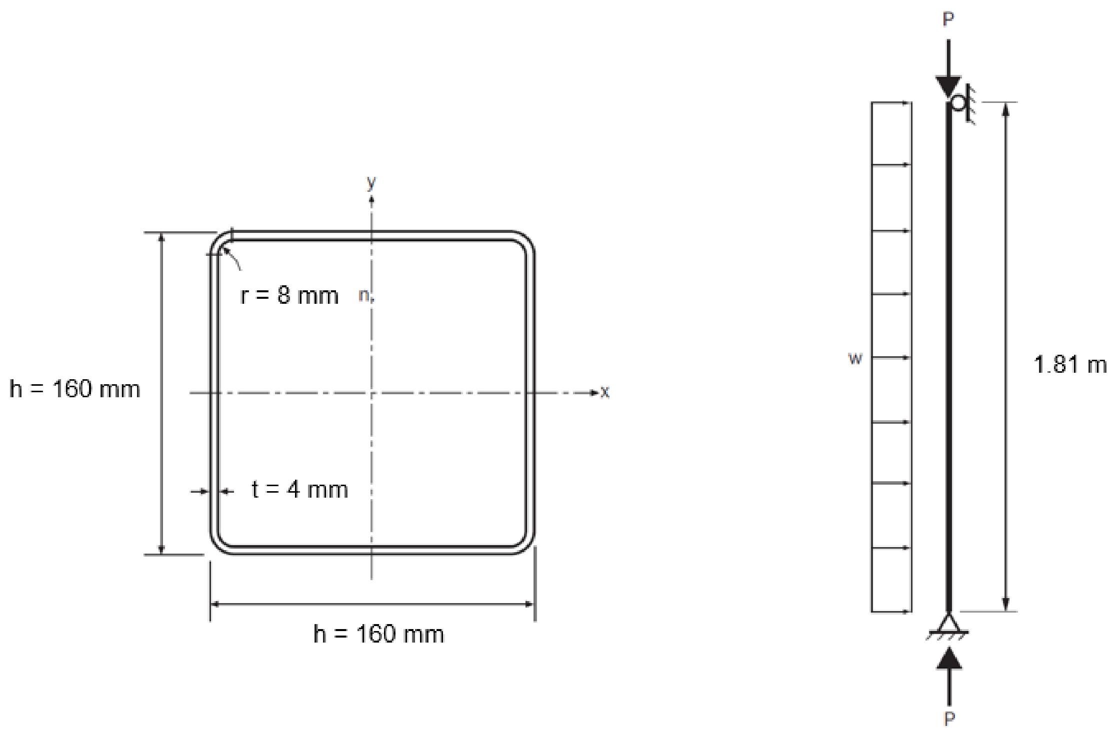
# EC3 1-3 2006 CFFD Example 003

## Box-Section Member Under Combined Compression, Bending, and Shear

### Example Description

Compression, moment, and shear capacities and demand/capacity ratio are calculated for box section at mid-height as shown below. It is simply supported with a length of 1.81 meters.

### Geometry, Properties and Loading




Dead: $P = 400{,}000\ N,\ w = 20\ N/mm$

### Technical Features Tested

- Axial compressive strength
- Major moment strength
- Shear strength
- Demand/Capacity ratio.

**COMPUTER FILE: EC3 1-3 2006 CFFD EX003**

## Applicable Programs

- SAP2000

## RESULTS COMPARISON

Independent results are hand calculated.

## CONCLUSION

The results show exact match with independent results.

## Benchmarks: SAP2000

| Output Parameter | Program | Independent | Percent Difference |
|---|---|---|---|
| Axial - Flexural buckling $N_{b,Rd}$ $(N)$ | 731486 | 731477 | 0.00% |
| Axial – Local & Distortional Buckling $N_{c,Rd}$ $(N)$ | 776251 | 776243 | 0.00% |
| Flexure – Lateral-Torsional Buckling $M_{b,Rd}$ $(N-mm)$ | 42501809 | 42501807 | 0.00% |
| Flexure – Local & Distortional Buckling $M_{c,Rd}$ $(N-mm)$ | 42501809 | 42501807 | 0.00% |
| Shear $V_{b,Rd}$ $(N)$ | 256963 | 256963 | 0.00% |
| D/C Ratio | 0.885 | 0.885 | 0.00% |

## HAND CALCULATION

**Properties:**

Material: $E = 210{,}000\ N/mm^2$, $G = 80{,}770\ N/mm^2$, $f_{yb} = 355\ N/mm^2$

Section:

$h = b = 160\ mm$, $t = 4\ mm$, $r = 8\ mm$

$\rightarrow h_p = b_p = h - t = 160 - 4 = 156\ mm$

Check for the effect of rounding of the corners:

$\frac{r}{t} = \frac{8}{4} = 2 < 5 \rightarrow OK$

$\frac{r}{b_p} = \frac{8}{156} = 0.051 < 0.1 \rightarrow OK$

Therefore, the effect of rounding of the corners can be neglected in calculation of section properties:

$A_g = 2496\ (mm^2)$

$I_y = I_z = 10122763.6\ (mm^4)$

$i_y = i_z = 63.684\ (mm)$

$W_{el} = 129779\ (mm^3)$

$I_t = 15185664\ (mm^4)$

$y_0 = z_0 = 0.0\ (mm)$

Member: $K_y = K_z = 0.9$ to model sway condition according to Annex BB.1.3(1) of EN1993-1-1

$L_y = L_z = 1810\ mm$

$k_{yy} = k_{zz} = k_{zy} = k_{yz} = 1.0$

**Loadings:** Dead: $P = 400{,}000\ N,\ w = 20\ N/mm$

**Required strengths:** for the section in the middle

$N_{Ed} = P = 400{,}000\ (N)$

$M_{Ed} = \frac{20 \times 1810^2}{8} = 8190250\ (N - mm)$

$V_{Ed} = 0.0\ (N)$

**Member Compression Capacity:** the compression capacity is calculated considering the limit states of global buckling, and local buckling. Distortional buckling is not considered because it is closed-box section.

1. Local buckling:
   The effective width method is utilized to calculate the nominal axial strength in consideration of local buckling with the compressive stress of $f_{yb} = 355\ (N/mm^2)$.

   Check for the applicability of the method as the following conditions are satisfied:

   $$\frac{h}{t} = \frac{b}{t} = \frac{160}{4} = 40 < 60 \rightarrow OK$$

   As the section is subjected to uniform compression and all four (4) sides are identical, they are considered internal stiffened elements:

   $$\psi = 1$$
   $$k_\sigma = 4$$
   $$\varepsilon = \sqrt{\frac{235}{f_{yb}[N/mm^2]}} = \sqrt{\frac{235}{355}} = 0.8136$$
   $$\bar{\lambda}_{p,b} = \frac{b_p/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{156/4}{28.4 \times 0.8136\sqrt{4}} = 0.844 > 0.673$$
   $$\rho = \frac{\bar{\lambda}_{p,b} - 0.055(3+\psi)}{\bar{\lambda}_{p,b}^2} = \frac{0.844 - 0.055(3+1)}{0.844^2} = 0.876 \le 1.0$$
   $$b_{eff} = \rho b_p = 0.876 \times 156 = 136.66\ (mm)$$
   $$A_{eff} = 4tb_{eff} = 4 \times 4 \times 136.66 = 2186.6\ (mm^2)$$
   $$A_{eff} = 2186.6\ (mm^2) < 2496\ (mm^2) = A_g$$
   $$\rightarrow N_{c,Rd} = \frac{A_{eff} f_{yb}}{\gamma_{M0}} = \frac{2186.6 \times 355}{1.0} = 776243\ (N)$$

   Because the section is doubly-symmetric, its effective properties are also symmetric about y-y and z-z axis, resulting in $e_{Ny} = e_{Nz} = 0 \rightarrow \Delta M_{y,Ed} = \Delta M_{z,Ed} = 0$

2. Global buckling: includes flexural buckling. Torsional and flexural-torsional buckling is not considered because it is box section
   i. Flexural buckling: since the member is doubly symmetric, flexural buckling strength is identical for y-y and z-z axis.

   $$N_{cr,y} = \frac{\pi^2 EI_y}{\left(K_y L_y\right)^2} = \frac{\pi^2(210{,}000)10122763.6}{(0.9 \times 1810)^2} = 7906348.8\ (N)$$
   $$\bar{\lambda}_y = \sqrt{\frac{A_{eff} f_{yb}}{N_{cr,y}}} = \sqrt{\frac{2186.6 \times 355}{7906348.8}} = 0.313$$

   For box section, the buckling curve is $c$ and $\alpha = 0.49$

$$\Phi_y = 0.5\left[1+\alpha\left(\bar{\lambda}_y - 0.2\right)+\bar{\lambda}_y^2\right] = 0.5[1+0.49(0.313-0.2)+0.313^2] = 0.577$$

$$\chi_y = \frac{1}{\Phi_y+\sqrt{\Phi_y^2-\bar{\lambda}_y^2}} = \frac{1}{0.577+\sqrt{0.577^2-0.313^2}} = 0.94233$$

$$N_{b,Rd} = \frac{\chi_y A_{eff} f_{yb}}{\gamma_{M1}} = \frac{0.94233\times 2186.6\times 355}{1.0} = 731477\ (N)$$

**Member Flexural Capacity:** the flexural capacity is calculated considering the limit state of local buckling. Lateral-torsional and distortional buckling are not considered because it is closed-box section.

1. Local buckling:

   The effective width method is utilized to calculate the nominal flexural strength in consideration of local and distortional buckling with the compressive stress in the top flange of $f_{yb} = 355\ (N/mm^2)$. As the section is subjected to positive moment, the top flange is under compression and it is considered as internal stiffened element:

$$\psi = 1$$

$$k_\sigma = 4$$

$$\varepsilon = \sqrt{\frac{235}{f_{yb}[N/mm^2]}} = \sqrt{\frac{235}{355}} = 0.8136$$

$$\bar{\lambda}_{p,b} = \frac{b_p/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{156/4}{28.4\times 0.8136\sqrt{4}} = 0.844 > 0.673$$

$$\rho = \frac{\bar{\lambda}_{p,b}-0.055(3+\psi)}{\bar{\lambda}_{p,b}^2} = \frac{0.844-0.055(3+1)}{0.844^2} = 0.876 \le 1.0$$

$$b_{eff} = \rho b_p = 0.876\times 156 = 136.66\ (mm)$$

   The bottom flange is in tension and calculation of its effective width is not carried out.
   The neutral axis of the section with effective flange width and gross web is:

$$\bar{y} = \frac{\sum_i A_i y_i}{A} = \frac{2th\frac{h}{2}+tbh}{t\left(b_{eff}+b+2h\right)} = \frac{2\times 4\times 156\frac{156}{2}+4\times 156\times 156}{4(136.66+156+2\times 156)} = 80.494\ (mm)$$

   The web is considered as internal (stiffened) element under stress gradient:

$$\sigma_1 = f_{yb} = 355\ (N/mm^2)$$

$$\sigma_2 = -f_{yb}\frac{156-80.494}{80.494} = -333\ (N/mm^2)$$

$$\psi = \frac{\sigma_2}{\sigma_1} = -\frac{333}{355} = -0.938$$

$$k_\sigma = 7.81-6.29\psi+9.78\psi^2 = 7.81-6.29\times(-0.938)+9.78\times(-0.938)^2 = 22.315$$

$$\bar{\lambda}_{p,b} = \frac{h_p/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{156/4}{28.4\times 0.8136\sqrt{22.315}} = 0.357 < 0.673 \rightarrow \rho = 1.0$$

$$h_c = \frac{h_p}{(1-\psi)} = \frac{156}{(1+0.938)} = 80.5\ (mm)$$
$$h_t = h_p - h_c = 156 - 80.5 = 75.5\ (mm)$$
$$h_{eff} = \rho \frac{h_p}{(1-\psi)} = 1.0 \times \frac{156}{(1+0.938)} = 80.5\ (mm)$$
$$h_{e1} = 0.4 h_{eff} = 0.4 \times 80.5 = 32.2\ (mm)$$
$$h_{e2} = 0.6 h_{eff} = 0.6 \times 80.5 = 48.3\ (mm)$$

The neutral axis of the section with effective top flange and lip measured from the centerline of the top flange is:

$$\bar{y} = \frac{\sum_i A_i y_i}{A} = 80.5\ (mm)$$
$$W_{eff,c} = 119723.4\ (mm^4)$$
$$W_{eff,t} = 127643.1\ (mm^4)$$
$$M_{c,Rd} = \frac{W_{eff,c} f_{yb}}{\gamma_{M0}} = \frac{119723.4 \times 355}{1.0} = 42501807\ (N-mm)$$

Lateral-torsional buckling is not required for box section, so $\chi_{LT}$ is taken as unity. The member bucklng moment strength is:

$$M_{b,Rd} = \chi_{LT} W_{eff,c} \frac{f_{yb}}{\gamma_{M1}} = 1.0 \times 119723.4 \frac{355}{1.0} = 42501807\ (N-mm)$$

**Member Shear Capacity:**

$$\bar{\lambda}_w = 0.346 \frac{s_w}{t} \sqrt{\frac{f_{yb}}{E}} = 0.346 \frac{156}{4} \sqrt{\frac{355}{210000}} = 0.555 < 0.83$$
$$f_{bv} = 0.58 f_{yb} = 0.58 \times 355 = 205.9\ (N/mm^2)$$
$$V_{b,Rd} = \frac{2 h_w t f_{bv}}{\gamma_{M0}} = \frac{2 \times 156 \times 4 \times 205.9}{1.0} = 256963\ (N)$$

**Combined D/C ratio:**

By observation, Equation 6.36 in Eurocode 3 1-3 2006 would provide the largest D/C ratio and govern the design

Eq. 6.36 in EC3 1-3:

$$\frac{D}{C} = \left(\frac{N_{Ed}}{N_{b,Rd}}\right)^{0.8} + \left(\frac{M_{y,Ed} + \Delta M_{y,Ed}}{M_{by,Rd}}\right)^{0.8} + \left(\frac{M_{z,Ed} + \Delta M_{z,Ed}}{M_{bz,Rd}}\right)^{0.8}$$
$$= \left(\frac{400{,}000}{731477}\right)^{0.8} + \left(\frac{8190250 + 0}{42501807}\right)^{0.8} + 0.0^{0.8} = 0.885$$